

OF THE
Laws of Chance,

OR, A
M E T H O D

Calculation of the Hazards

G A M E,

Plainly demonstrated,

And applied to **G A M E S** at
present most in Use,

Which may be easily extended to the most
intricate Cases of Chance imaginable.

L O N D O N:

Printed by *Benj. Motte*, and sold by
Randall Taylor near *Stationers-Hall*, 1692.

OF THE

LAW OF CHURCH

OF A



MVSEVM
BRITAN
NICVM

British Museum

of Natural History

London

Printed by J. G. Smith, 1851

P R E F A C E.

IT is thought as necessary to write a Preface before a Book, as it is judg'd civil, when you invite a Friend to Dinner, to proffer him a Glass of Hock before-hand for a Whet: And this being maim'd enough for want of a Dedication, I am resolv'd it shall not want an Epistle to the Reader too. I shall not take upon me to determine, whether it is lawful to play at Dice or not, leav-

Preface.

ing that to be disputed betwixt the Fanatick Parsons and the Sharpers; I am sure it is lawful to deal with playing at Dice as with other Epidemic Distempers; and I am confident that the writing a Book about it, will contribute as little towards its Encouragement, as Fluxing and Precipitates do to Whoring.

It will be to little purpose to tell my Reader, of how great Antiquity the playing at Dice is, I will only let him know,

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Preface.

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know, that by the Alex Ludus, the Antients comprehended all Games, which were subjected to the determination of mere Chance; this sort of Gaming was strictly forbid by the Emperor Justinian, Cod. Lib. 3. Tit. 42. under very severe Penalties; and Photius Nomocan, Tit. 9. Cap. 27. acquaints us, that the Use of this was altogether denied the Clergie of that time. Seneca says very well, Aleator quantò in arte est melior, tantò est ne-
A 3 quior;

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quior ; *That by how much the one is more skilful in Games, by so much he is the more culpable ; or we may say of this, as an ingenious Man says of Dancing, That to be extraordinary good at it, is to be excellent in a Fault ; therefore I hope no body will imagine I had so mean a Design in this, as to teach the Art of Playing at Dice.*

A great part of this Discourse is a Translation from Mons. Hugens's Treatise, De ratiociniis in ludo Alex,

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Alex, one, who in his Improvements of Philosophy, has but one Superior, and I think few or no Equals. The whole I undertook for my own Divertisement, next to the Satisfaction of some Friends, who would now and then be wrangling about the Proportions of Hazards in some Cases that are here decided. All it requir'd was a few spare Hours, and but little Work for the Brain; my Design in publishing it, was to make it of more general
A 4 Use,

Preface.

Use, and perhaps persuade
a ~~raw~~ Squire, by it, to keep
his Money in his Pocket;
and if, upon this account, I
should incur the Clamours of
the Sharpers, I do not much
regard it, since they are a
sort of People the World is
not bound to provide for.

You will find here a very
plain and easie Method of the
Calculation of the Hazards of
Game, which a man may un-
derstand, without knowing the
Quadratures of Curves, the
Doctrin of Series's, or the
Laws

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Laws of Centripetation of Bodies, or the Periods of the Satellites of Jupiter ; yea, without so much as the Elements of Euclid. There is nothing required for the comprehending the whole, but common Sense and practical Arithmetick ; saving a few Touches of Algebra, as in the first Three Propositions, where the Reader, without Suspicion of Popery, may make use of a strong implicit Faith ; tho I must confess, it does not much recommend

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*it self to me in these purposes ;
for I had rather he would en-
quire, and I believe he will find
the speculation not unpleasant.*

*Every man's Success in any
Affair is proportional to his
Conduct & Fortune. Fortune
(in the sense of most People)
signifies an Event which de-
pends on Chance, agreeing with
my Wish; and Misfortune sig-
nifies such an Event contrary
to my Wish : an Event depen-
ding on Chance, signifies such
an one, whose immediate Causes
I don't know, and consequently*

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can neither foretel nor produce it (for it is no Heresie to believe, that Providence suffers ordinary matters to run in the Channel of second Causes). Now I suppose, that all a wise Man can do in such a Case is, to lay his Business on such Events, as have the most or most powerful second Causes, and this is true both in the great Events of the World, and in ordinary Games. It is impossible for a Dye, with such a determin'd force and direction, not

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to fall on such a determin'd side, only I don't know the force and direction which makes it fall on such a determin'd side, and therefore I call that Chance, which is nothing but want of Art; that only which is left to me, is to wager where there are the greatest number of Chances, and consequently the greatest probability to gain; and the whole Art of Gaming, where there is any thing of Hazard, will be reduc'd to this at last, viz. in dubious Cases, to calculate

Preface.

calculate on which side there are most Chances ; and tho this can't be done in the midst of Game precisely to an Unite, yet a Man who knows the Principles, may make such a conjecture, as will be a sufficient direction to him ; and tho it is possible, if there are any Chances against him at all, that he may lose, yet when he chuseth the safest side, he may part with his Money with more content (if there can be any at all) in such a Case.

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I will not debate, whether one may engage another in a disadvantageous Wager. Games may be suppos'd to be a tryal of Wit as well as Fortune, and every Man, when he enters the Lists with another, unless out of Complaisance, takes it for granted, his Fortune and Judgment, are, at least, equal to those of his Play-Fellow; but this I am sure of, that false Dice, Tricks of Leger-de-main, &c. are inexcusable, for the question in Gaming

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ming is not, *Who is the best
Fugler?*

The Reader may here observe the Force of Numbers, which can be successfully applied, even to those things, which one would imagine are subject to no Rules. There are very few things which we know, which are not capable of being reduc'd to a Mathematical Reasoning, and when they cannot, its a sign our Knowledge of them is very small and confus'd; and where

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a mathematical reasoning can be had, it's as great folly to make use of any other, as to grope for a thing in the dark when you have a Candle standing by you. I believe the Calculation of the Quantity of Probability might be improved to a very useful and pleasant Speculation, and applied to a great many Events which are accidental, besides those of Games; only these Cases would be infinitely more confus'd, as depending on Chances which the most part of Men

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Men are ignorant of ; and as I have hinted already, all the Politicks in the World are nothing else but a kind of Analysis of the Quantity of Probability in casual Events, and a good Politician signifies no more, but one who is dexterous at such Calculations ; only the Principles which are made use of in the Solution of such Problems, can't be studied in a Closet, but acquir'd by the Observation of Mankind.

There

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There is likewise a Calculation of the Quantity of Probability founded on Experience, to be made use of in Wagers about any thing ; for Example, it is odds, if a Woman is with Child, but it shall be a Boy ; and if you would know the just odds, you must consider the Proportion in the Bills that the Males bear to the Females : The Yearly Bills of Mortality are observ'd to bear such Proportion to the live People as 1 to 30, or 26 ; therefore it is

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is an even Wager, that one out of thirteen, dyes within a Year (which may be a good reason, tho not the true one of that foolish piece of Superstition), because, at this rate, if 1 out of 26 dyes, you are no loser. It is but 1 to 18 if you meet a Parson in the Street, that he proves to be a Non-Juror, because there is but 1 of 26 that are such. It is hardly 1 to 10, that a Woman of Twenty Years old has her Maidenhead, and almost the

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the same Wager, that a Town-Spark of that Age has not been clap'd. I think a Man might venture some odds, that 100 of the Gens d'arms beats an equal Number of Dutch Troopers; and that an English Regiment stands its ground as long as another, making Experience our Guide in all these Cases and others of the like nature.

But there are no casual Events, which are so easily subjected to Numbers, as those

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those of Games; and I believe, there the Speculation might be improved so far, as to bring in the Doctrine of the Series's and Logarithms. Since Gaming is become a Trade, I think it fit the Adventurers should be upon the Square; and therefore in the Contrivance of Games there ought to be a strict Calculation made use of, that they mayn't put one Party in more probability to gain than another; and likewise, if a Man has a
con-

Preface.

considerable Venture, he ought to be allow'd to withdraw his Money when he pleases, paying according to the Circumstances he is then in: and it were easie in most Games to make Tables, by the Inspection of which, a Man might know what he was either to pay or receive, in any Circumstances you can imagin, it being convenient to Save a part of ones Money, rather than venture the loss of it all.

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I shall add no more, but that a Mathematician will easily perceive, it is not put in such a Dress as to be taken notice of by him, there being abundance of Words spent to make the more ordinary sort of People understand it.

For

FOR the sake of those who are not vers'd in *Mathematicks*, I have added the following Explanation of Signs.

= *Equal.*

+ *More, or to be added.*

- *Less, or to be subtracted.*

x *Multiplied.*

Example.

$3 \times 4 + 3 - 1 = 14 = \frac{2}{9} a$, is to be read thus,

3 multiplied in 4 more by 3 less by 1 is equal to 14, which is equal to five ninth parts of a .

An Exact
METHOD

For SOLVING the
Hazards of Game.

Although the Events
of Games, which
Fortune solely go-
vern, are uncer-
tain, yet it may be certainly de-
termin'd, how much one is
more ready to lose than gain.
For Example: If one should
wager, at the first Throw with
one Dye, to throw Six, it's an
B accident

accident if he gains or not, but by how much it's more probable he will lose than gain, is really determin'd by the Nature of the thing, and capable of a strict Calculation. So likewise, if I should play with another on this condition, that the Victory should be to the Three first Games, and I had gain'd one already; it is still uncertain who shall first gain the third; yet by a demonstrative reasoning I can estimate both the Value of his expectation and mine, and consequently (if we agree to leave the Game imperfect) determine how great a share of the Stakes belong to me, and how much to my Play-fellow;
or

or if any were desirous to take my place, at what rate I ought to sell it. Hence may arise innumerable Queries among two, three, or more Gamesters; and since the Calculation of these things is a little out of the common road, and can be oft-times apply'd to good purpose; I shall briefly here shew how it is to be done, and afterwards explain those things which belong properly to the Dice.

In both cases I shall make use of this Principle, *Ones Hazard or Expectation to gain any thing, is worth so much, as, if he had it, he could purchase the like Hazard or Expectation again in a just and equal Game.*

4 A Solution of the

For Example, If one, without my knowledg, should hide in one Hand 7 Shillings, and in his other 3 Shillings, and put it to my choice which Hand I would take, I say this is as much worth to me, as if he should give me 5 Shillings; because, if I have 5 Shillings, I can purchase as good a Chance again, and that in a fair and just Game.

PRO-

PROPOSITION I.

If I expect a or b , either of which, with equal probability, may fall to me, then my Expectation is worth $\frac{a+b}{2}$, that is, the half Sum of a and b .

That I may not only demonstrate, but likewise investigate this Rule, suppose the Value of my Expectation be x ; by the former Principle having x , I can purchase as good an Expectation again in a fair and just Game. Suppose then I play with another on these

B 3 terms;

terms; That every one stakes x , and the Gainer give to the Loser a , this Game is just, and it appears, that at this rate, I have an equal hazard either to get a if I lose the Game, or $2x - a$ if I gain; for in this case I get $2x$, which are the Stakes, out of which I must pay the other a ; but if $2x - a$ were worth b , then I have an equal hazard to get a or b ; therefore making $2x - a = b$,

$$x = \frac{a + b}{2}, \text{ which is the Value}$$

of my Expectation. The Demonstration is easie, for having

$$\frac{a + b}{2}, \text{ I can play with another}$$

who

who will stake $\frac{a+b}{2}$ against it, on this condition, that the Gainer should give to the Loser a ; by this means I have an equal Expectation to get a if I lose, or b if I win; for in the last case I get $a+b$ the Stakes, out of which I must pay a to my Play-fellow.

In Numbers: If I had an equal hazard to get 3 or 7, then by this Proposition, my Expectation is worth 5, and it is certain, having 5, I may have the same Chance; for if I play with another so that every one stakes 5, and the Gainer pay to the Loser 3, this is a fair way of Gaming;

B 4 and

and it is evident, I have an equal hazard to get 3 if I lose, or 7 if I gain.

P R O P. II.

If I expect a , b , or c , either of which, with equal facility, may happen, then the Value of my Expectation is $\frac{a + b + c}{3}$, or the third part of the Sum of a b and c .

FOR the Investigation of which, suppose x be the value of my Expectation; then x must be such, as I can purchase with it the same Expectation in a just Game: Suppose the

Hazards of Game.

9

the Conditions of the Game be, that playing with two others, each of us stakes x , and I bargain with one of the Gamesters, if I win, to give him b , and he shall do the same to me; but with the other, that if I gain, I shall give him c , and *vice versa*; this is fair play: And here I have an equal hazard to get b , if the first win, c if the second, or $3x - b - c$ if I gain my self; for then I get $3x$, *viz.* the Stakes, of which I give the one b and the other c ; but if $3x - b - c$ be equal to a , I have an equal Expectation of a , b , or c ; therefore making $3x - b - c = a$,

B 5

 $x =$

10 Solution of the

$x = \frac{a+b+c}{3}$, which is the Value of my Expectation. After the same method you will find, if I had an equal hazard to get $a b c$ or d , the Value of my Expectation $\frac{a+b+c+d}{4}$ that is the fourth part of the Sum of $a b c$ and d , &c.

PROP. III.

If the Number of Chances, by which a falls to me, be p , and the Number of Chances, by which b falls, be q , and supposing all the Chances do happen with equal Facility, then

Hazards of Game. 11

then the Value of my Expectation is $\frac{pa + bq}{p + q}$, i. e.

the Product of a multiplied in the Number of its Chances added to the Product of b , multiplied into the Number of its Chances, and the Summ divided by the Number of Chances both of a and b .

Suppose, as before, x be the Value of my Expectation; then if I have x , I must be able to purchase with it that same Expectation again in a fair Game: For this I shall take as many Play-fellows as, with me, make up the Number

ber

ber of $p + q$, of which let every one stake x , so the whole Stake will be $px + qx$, and every one plays with equal hopes of winning; with as many of my Fellow-Gamesters as the Number q stands for, I make this bargain one by one, that whoever of them gains shall give me b , and if I win, I shall do so to them; with every one of the rest of the Gamesters, whose Number is $p - 1$, I make this bargain, that whoever of them gains, shall give me a , and I shall give every one of them as much, if I gain: It's evident this is fair play; for no Man here is injur'd; and in this case I have q Expectations to

to gain b , and $p - 1$ Expectations to gain a , and 1 Expectation (*viz.* when I win my my self) to get $px + qx - bq - ap + a$; for then I am to deliver b to every one of the q Players, and a to every one of the $p - 1$ Gamesters, which makes $ab + pa - a$; if therefore $qx + bx - ba - ap + a$ were equal to a , I would have p Expectations of a (since just now I had $p - 1$ Expectations of it) and q Expectations of b , and so would have just come to my first Expectation; therefore putting $px + qx - bq - ap + a = a$, then is $x = \frac{ap + bq}{p + q}$.

14 *Solution of the*

In *Numbers*: If I had 3 Chances to gain for 13, and 2 for 8, by this Rule, my Hazard is worth 11; for 13 multiplied by 3 gives 39, and 8 by 2 16, these two added, make 55, divided by 5 is 11, and I can easily shew, if I have 11, I can come to the like Expectation again; for playing with four others, and every one of us staking 11, with two of them I make this Bargain, that whoever gains shall give me 8, and I shall too do so to them; with the other two I make this Bargain, that whoever gains shall give me 13, and I them as much if I gain: It appears, by this means I have two Expectations

Hazards of Game. 15

tions to get 8, viz. if any of the first two gain, and 3 Expectations to get 13, viz. if either I or any of the other two gain; for in this case I gain the Stakes, which are 55, out of which I am oblig'd to give the first two 8, and the other two 13, and so there remains 13 for my self.

P R O P. IV.

That I may come to the Question propos'd, viz. The making a just Distribution amongst Gamesters, when their Hazards are unequal; we must begin with the most easie Cases.

Suppose

Suppose then I play with another, on condition that he who wins the three first Games shall have the Stakes; and that I have already gain'd two, I would know, if we agree to break off the Game, and part the Stakes justly, how much falls to my share?

The first thing we must consider in such Questions is, the Number of Games that are wanting to both: For *Example*, If it had been agreed betwixt us, that he should have the Stakes who gain'd the first 20 Games, and if I had gain'd already 19, and my Fellow-Gamester but 18, my Hazard
is

Hazards of Game. 17

is as much better than his in that Case, as in this proposed, *viz?* When of 3 Games I have 2, and he but 1, because in both cases there's 2 wanting to him, and 1 to me.

In the next place, to find the portion of the Stakes due to each of us, we must consider what would happen if the Game went on; it is certain, if I gain the first Game, I get the Stake, which I call a ; but if he gain'd, both our Lots would be equal, and so there would fall to each of us $\frac{1}{2}a$; but since I have an equal Hazard to gain or lose the first Game, I have an equal Expectation to gain a , or $\frac{1}{2}a$, which, by the first Proposition, is as much worth

worth as the half Sum of both,
i. e. $\frac{3}{4}a$, so there is left to my
 Fellow - Gamester $\frac{1}{4}a$; from
 whence it follows, that he who
 would buy my Game, ought
 to pay me for it $\frac{3}{4}a$; and there-
 fore, he who undertakes to gain
 one Game before another gains
 two, may wager 3 to 1.

PROP. V.

*Suppose I want but one Game,
 and my Fellow-gamester three,
 it is required to make a just
 Distribution of the Stake:*

LET us here likewise con-
 sider in what state we
 should be, if I or he gain'd the
 first

first Game; if I gain, I have the Stake a , if he, then he wants yet 2 Games, and I but 1, and therefore we should be in the same condition which is supposed in the former Proposition; and so there would fall to my share, as was demonstrated there, $\frac{3}{4}a$; therefore with equal facility there may happen to me a , or $\frac{3}{4}a$, which, by the First Proposition, is worth $\frac{7}{8}a$, and to my Fellow-Gamester there is left $\frac{1}{8}a$, and therefore my Hazard to his is as 7 to 1.

As the Calculation of the former Proposition was requisite for this, so this will serve for the following. If I should suppose my self to want but
one

one Game, and my Fellow four (by the same Method) you will find $\frac{15}{16}$ of the Stake belongs to me, and $\frac{1}{16}$ to him.

P R O P. VI.

Suppose I want two Games, and my Fellow-Gamester three.

Then by the next Game it will happen, that I want but one, and he three; which (by the preceding Proposition) is worth $\frac{7}{8}a$; or that we should both want two, whence there will be $\frac{1}{2}a$ due to each of us; now I being in an equal probability to gain or lose the next Game, I have an equal Hazard
to

to gain $\frac{7}{8}a$ or $\frac{1}{2}a$, which, by the First Proposition is worth $\frac{11}{16}a$, and so there are eleven parts of the Stakes due to me, and five to my Fellow.

P R O P. VII.

Let us suppose I want two Games, and my Fellow four.

IF I gain the next Game, then I shall want but one, and my Fellow four; but if I lose it, then I shall want two, and he three: So I have an equal Hazard for gaining $\frac{15}{16}a$, or $\frac{11}{16}a$, which, by the First, is worth $\frac{13}{16}a$: So it appears, that he who is to gain two Games for the others

thers four, is in a better condition than he who is to gain one for the others two; for my share in the first Case is $\frac{3}{4}a$ or $\frac{12}{16}a$, which is less than $\frac{13}{16}$ my share in the last.

P R O P. VIII.

Let us suppose three Gamesters, whereof the first and second want 1 Game, but the third 2.

TO find the share of the first, we must consider what would happen if either he, or any of the other two gain'd the first Game; if he gains, then he has the Stake a ; if the second

cond gain, he has nothing; but if the third gain, then each of them would want a Game, and so $\frac{1}{3}a$ would be due to every one of them. Thus the first Gamester has one Expectation to gain a , one to gain nothing, and one for $\frac{1}{3}a$ (since all are in an equal probability to gain the first Game) which by the second Proposition is worth $\frac{4}{9}a$: Now since the second Gamesters Condition is as good, his Share is likewise $\frac{4}{9}a$, and so there remains to the third $\frac{1}{9}a$, whose Share might have been as easily found by its self.

PROP.

P R O P. IX.

In any Number of Gamesters you please, amongst whom there are some who want more, some fewer Games: To find what is any ones Share in the Stake, we must consider, what would be due to him, whose Share we investigate, if either he, or any of his Fellow-Gamesters should gain the next following Game; add all their Shares together, and divide the Sum by the Number of the Gamesters, the Quotient is his Share you were seeking.

Suppose

Suppose three Gamesters, A B and C, A wants 1 Game, B 2, and C likewise 2, I would find what is the Share of the Stake due to B, which I shall call q .

First we must consider what would fall to B's share, if either he, A, or C, wins the next Game; if A wins, the Game is ended, so he gets nothing; if B himself gain, then he wants 1 Game, A 1, and C 2; therefore, by the former Proposition, there is due to him in that Case q , then if C gains the next Play, then A and C would want but 1, and B 2; and therefore, by the Eighth Propo-
C
sition,

sition, his Share would be worth $\frac{1}{9}q$; add together what is due to B in all these three Cases, viz. $0\frac{4}{9}q$, $\frac{1}{9}q$, the Sum is $\frac{5}{9}q$, which being divided by 3, the Number of Gamesters gives $\frac{5}{27}q$, which is the Share of B sought for: The Demonstration of this is clear from the Second Proposition, because B has an equal Hazard to gain $0\frac{4}{9}q$ or $\frac{1}{9}q$, that is, $\frac{0\frac{4}{9} + \frac{1}{9}q}{3}$, i. e. $\frac{5}{27}q$;

now it's evident the Divisor 3 is the Number of the Gamesters.

To find what is due to one in any Case, viz. if either he, or any of his Fellow-Gamesters win the following Game; we

we must consider first the more simple Cases, and by their help the following; for as this Case could not be solv'd before the Case of the Eighth Proposition was calculated, in which, the Games wanting were 1, 1, 2; so the Case, where the Games wanting are 1, 2, 3, cannot be calculated, without the Calculation of the Case, where the Games wanting are 1, 2, 2, (which we have just now perform'd) and likewise of the Case, where the Games wanting are 1, 1, 3, which can be done by the Eighth: And by this means you may reckon all the Cases comprehended in the following Tables, and an infinite number of others.

Games wanting	1, 1, 2,	1, 2, 2, 1,	1, 1, 3	1, 2, 3,
Their Shares.	4, 4, 1,	17, 5, 5,	13, 13, 1,	19, 6, 2,
	9	27	27	27

Gam. want.	1, 1, 4	1, 1, 5	1, 2, 4	1, 2, 5
Shares.	40, 40, 1	121, 121, 1	178, 58, 7	542, 179, 8
	81	243	243	729

Games wanting.	1, 3, 3	1, 3, 4	1, 3, 5
Their Shares.	65, 8, 8	616, 82, 31	629, 87, 13
	81	729	729

Gam. want.	2, 2, 3	2, 2, 4	2, 2, 5
Their Shares.	34, 34, 13	338, 338, 53	353, 353, 23
	81	729	729

G. want.	2, 3, 3	2, 3, 4	2, 3, 5
Shares.	133, 55, 55	451, 195, 83	433, 635, 119
	243	729	2187

As for the *Dice* ; these Questions may be proposed, at how many Throws one may wager to throw 6, or any Number below that, with one Dye ; How many Throws are required for 12 upon two Dice ; or 18 on 3 ; and several other Questions to this purpose.

For the resolving of which, it must be consider'd, that in one Dye there are six different Throws, all equally probable to come up; for I suppose the Dye has the exact figure of a Cube : On Two Dice there are 36 different Throws ; for in respect to every Throw of One Dye, any One Throw of the 6 of the other Dye may come up ; and 6 times

C 3 6 make

6 make 36: In Three Dice there are 216 different Throws; for in relation to any of the 36 Throws of Two Dice, any one of the six of the Third may come up; and 6 times 36 make 216: So in Four Dice there are 6 times 216 Throws, that is, 1296: And so forward you may reckon the Throws of any Number of Dice, taking always, for the addition of a new Dye, 6 times the Number of the preceeding.

Besides, it must be observ'd, that in Two Dice there is only one way 2 or 12 can come up; two ways that 3 or 11 can come up; for if I shall call the Dice A and B to make 3, there
may

Hazards of Game. 31

may be 1 in A and 2 in B, or 2 in A and 1 in B; so to make 11, there may be 5 in A or 6 in B, or 6 in A and 5 in B; for 4 there are three Chances, 3 in A and 1 in B, 3 in B and 1 in A, or 2 as well in A as B; for 10 there are likewise three Chances; for 5 or 9 there are four Chances; for 6 or 8 five Chances; for 7 there are six Chances.

In 3 Dice there are found for	3 or 18	1
	4 or 17	3
	5 or 16	6
	6 or 15	10
	7 or 14	15
	8 or 13	21
	9 or 12	25
	10 or 11	27

C 4

PROP.

PROP. X.

*To find at how many times one
may undertake to throw 6
with One Dye.*

IF any should undertake to throw 6 the first time, it's evident there's one Chance gives him the Stake, and five which give him nothing; for there are 5 Throws against him, and only 1 for him: Let the Stake be call'd a , then he has one Expectation to gain a , and five to gain nothing, which, by the Second Proposition, is worth $\frac{1}{6}a$, and there remains for the other $\frac{5}{6}a$; so he who undertakes, with

with one Dye, to throw 6 the first time, ought to wager only 1 to 5.

2. Suppose one undertake, at two Throws of 1 Dye, to throw 6, his Hazard is calculated thus ; if he throw 6 at the first he has a the Stake, if he do not, there remains to him one Throw, which, by the former Case, is worth $\frac{1}{6}a$; but there is but one Chance which gives him 6 at the first Throw, and five Chances against him ; so there is one Chance which gives him a , and five which give him $\frac{1}{6}a$, which, by the Second Proposition, is worth $\frac{11}{36}a$, so there remains to his Fellow-Gamester $\frac{25}{36}a$; so the Value of my Expectation to his, is

C 5 as

as 11 to 25, i. e. less than 1 to 2.

By the same method of calculation, you will find, that his Hazard who undertakes to throw 6 at three times with one Dye, is $\frac{91}{216}a$; so that he can only lay 91 against 125, which is something less than 3 to 4.

He who undertakes to do it at four times, his Hazard is $\frac{671}{1296}a$, so he may wager 671 against 625, that is, something more than 1 to 1.

He who undertakes to do it at five times, his Hazard is $\frac{4651}{7776}a$, so he can wager 4651 against 3125, that is something less than 3 to 2.

His

His Hazard who undertakes to do it 6 times, is $\frac{3 \cdot 231}{45656}a$, and he can wager 31031 against 15625, that is something less than 2 to 1.

Thus any Numb. of Throws may be easily found, but the following Proposition will shew you a more compendious way of Calculation.

PROP. XI.

To find at how many times one may undertake to throw 12 with Two Dice.

IF one should undertake it at One Throw, it's clear he has but one Chance to get the Stake

Stake a , and 35 to get nothing ;
 so, by the Second Proposition,
 he has much $\frac{1}{36}a$.

He who undertakes to do it
 at Twice, if he throw 12 the
 first time gains a , if otherwise,
 then there remains to him One
 Throw, which, by the former
 Case, is worth $\frac{1}{36}a$; but there is
 but One Chance which gives
 12 at the first Throw, and 35
 Chances against him ; so he has
 1 Chance for a , and 35 for $\frac{1}{36}a$,
 which, by the Second Proposi-
 tion, is worth $\frac{71}{1296}a$, and there
 remains to his Fellow-Gamester
 $\frac{1225}{1296}a$.

From these it's easie to find
 the Value of his Hazard, who
 undertakes it at four times ; pas-
 sing

sing by his Case who undertakes it at three times.

If he who undertakes to do it at four times throws 12 the first or second Cast, then he has a , if not, there remains two other Throws, which, by the former Case, are worth $\frac{71}{1296}a$; but for the same reason, in his two first Throws, he has 71 Chances which give him a , against 1225 Chances, in which it may happen otherwise; therefore at first he has 71 Chances which give him a , and 1225 which give him $\frac{71}{1296}a$, which, by the Second Proposition, is worth $\frac{15006:5}{1679516}a$, which shews that their Hazards to one another are as 178991 to 1500625.

From

From which Cases it is easie to find the Value of his Expectation, who undertakes to do it at 8 times, and from that, his Case who undertakes to do it at 16 times; and from his Case who undertakes to do it at 8 times; and his likewise who undertakes to do it at 16 times; it is easie to determin his Expectation who undertakes it at 24 times: In which Operation, because that which is principally sought, is the Number of Throws, which makes the Hazard equal on both sides, *viz.* to him who undertakes, and he who offers, you may, without any sensible Error, from the Numbers (which else would grow

grow very great) cut off some of the last Figures. And so I find, that he who undertakes to throw 12 with Two Dice, at 24 times, has some Loss, and he who undertakes it at 25 times, has some Advantage.

PROP. XII.

*To find with how many Dice,
one can undertake to throw
two Sixes at the first Cast.*

THis is as much, as if one would know, at how many Throws of one Dye, he may undertake to throw twice Six; now if any should undertake it at two Throws, by what

what we have shewn before, his Hazard would be $\frac{1}{36}a$, he who would undertake to do it at 3 ; times, if his first Throw were not 6, then there would remain two Throws, each of which must be 6, which (as we have said) is worth $\frac{1}{36}a$; but if the first Throw be 6, he wants only one 6 in the two following Throws, which by the Tenth Proposition, is worth $\frac{11}{36}a$; but since he has but one Chance to get 6 the first Throw, and five to miss it; he has therefore, at first, one Chance for $\frac{11}{36}a$, and five Chances for $\frac{1}{36}a$, which, by the Second Proposition, is worth $\frac{16}{216}a$, or $\frac{2}{27}a$, after this manner still assuming 1 Chance more,

more, you will find that you may undertake to throw two Sixes at 10 Throws of one Dye, or 1 Throw of ten Dice, and that with some Advantage.

P R O P. XIII.

If I am to play with another One Throw, on this condition, that if 7 comes up I gain, if 10 he gains; if it happens that we must divide the Stake, and not play, to find how much belongs to me, and how much to him.

BEcause of the 36 different Throws of the Two Dice, there are six which give 7 and

7 and 3, which give 10 and 27, which equals the Game, in which Case there is due to each of us $\frac{1}{2}a$: But if none of the 27 should happen, I have 6, by which I may gain a , and 3, by which I may get nothing, which, by the Second Proposition, is worth $\frac{2}{3}a$; so I have 27 Chances for $\frac{1}{2}a$, and 9 for $\frac{2}{3}a$, which, by the second Proposition, is worth $\frac{13}{24}a$, and there remains to my Fellow-Gamester $\frac{11}{24}a$.

PROP.

P R O P. XIV.

If I were playing with another by turns, with two Dice, on this condition, that if I throw 7 I gain, and if he throw 6 he gains, allowing him the first Throw : To find the proportion of my Hazard to his.

Suppose I call the Value of my Hazard x , and the Stakes a , then his Hazard will be $a - x$; then whenever it's his turn to throw, my Hazard is x , but when it's mine, the Value of my Hazard is greater. Suppose I then call it y ; now because of the 36 Throws of
Two

Two Dice, there are five which give my Fellow-Gamester 6, thirty one which bring it again to my turn to throw, I have five Chances for nothing, and thirty one for y , which, by the Third Proposition, is worth $\frac{31}{36}y$; but I suppos'd at first my Hazard to be x ; therefore $\frac{31}{36}y = x$, and consequently $y = \frac{36}{31}x$. I suppos'd likewise, when it was my turn to throw, the Value of my Hazard was y , but then I have six Chances which give me 7, and consequently the Stake, and thirty which give my Fellow the Dice, that is, make my Hazard worth x ; so I have six Chances for a , and thirty for x , which, by Prop.

Prop. 3. is worth $\frac{6a + 30x}{36}$

but this by supposition is equal to y , which is equal (by what has been prov'd already) to $\frac{36}{31}x$;

therefore $\frac{30x + 6a}{36} = \frac{36}{31}x$, and

consequently $x = \frac{31}{61}a$, the Value of my Hazard, and that of my Fellow-Gamester is $\frac{30}{61}a$; so that mine is to his as 31 to 30.

Here follow some Questions which serve to exercise the former Rules.

1. A and B play together with two Dice, A wins if he throws 6, and B if he throws 7; A at first gets one Throw, then B two, then A two, and so

so on by turns, till one of them wins. I require the proportion of A's Hazard to B's? *Answer*, It is as 10355 to 12276.

2. Three Gamesters, A, B, and C, take 12 Counters, of which there are four white and eight black; the Law of the Game is this, that he shall win, who, hood-wink'd, shall first chuse a white Counter, and that A shall have the first choice, B the second, and C the third, and so, by turns, till one of them win. *Quer.* What is the proportion of their Hazards?

3. A wagers with B, that of 40 Cards, that is, 10 of every

every Suit, he will pick out four ; so that there shall be one of every suit ; A's Hazard to B's, in this Case, is as 1000 to 8139.

4. Supposing, as before, 4 white Counters and 8 black, A wagers with B, that out of them, he shall pick 7 Counters, of which there are 3 white. I require the proportion of A's Hazard to B's ?

5. A and B taking 12 Counters, play with three Dice after this manner ; that if 12 comes up, A shall give one Counter to B, but if 14 comes up, B shall give one to A, and that
he

he shall gain who first has all
the Counters. A's Hazard to
B's is 244140625 to 282429
536481.

The *Calculus* of the preceed-
ing Problems is left out by
Mons. *Hugens*, on purpose that
the ingenious Reader may have
the satisfaction of applying the
former Method himself; it is in
most of them more laborious
than difficult; for *Example*, I
have pitch'd upon the Second
and Third, because the rest can
be solv'd after the same Method.

Prob.

Problem 1.

The first Problem is solv'd by the Method of Prop. 14. only with this difference, that after you have found the share due to B, if A were to get no first Throw, you must subtract from it $\frac{5}{36}$ of the Stake which is due to A for his Hazard of throwing Six at the first Throw.

Probl. 2

As for the second Problem, it is solved thus, Suppose A's Hazard, when it is his own turn to chuse, be x , when it is B's, be y , and when it is C's,
D be

50 *Solution of the*

be z ; it is evident, when out of 12 Counters, of which there are 4 white and 8 black, he endeavours to chuse a white one, he has four Chances to get it, and eight to miss it, that is, he has four Chances to get the Stake a , and eight to make his Hazard worth y ; so $x = \frac{4a + 8y}{12}$, and consequently $y =$

$$\frac{12x - 4a}{8}.$$

When it is B's turn to chuse, then he has four Chances for nothing, and eight for z , (that is to bring it to C's turn) consequently $y = \frac{8}{12} z =$

$$\frac{12x - 4a}{8};$$

this equation re-

duc'd

duc'd gives $z = \frac{9x - 3a}{4}$; when

it comes to C's turn to chuse then A has four Chances for nothing, and eight for x , consequently $z = \frac{8}{12}x$, therefore

$$\frac{8}{12}x = \frac{9x - 3a}{4}; \text{ this equation}$$

reduc'd gives $x = \frac{9}{19}a$, and consequently there remains to the B and C $\frac{10}{19}a$, which must be shar'd after the same manner, that is, so that B have the first Choice, C the next, and so on, till one of them gain; the reason is, because it had been just in A to have demanded $\frac{9}{19}$ of the Stake for not playing, and then the seniority fell to B;

now $\frac{10}{19}$ *a* parted betwixt B and C, by the former method, gives $\frac{6}{19}$ to B, and $\frac{4}{19}$ to C; so A, B, and C's Hazards, from the beginning, were as 9, 6, 4.

I have suppos'd here the sense of the Problem to be, that when any one chus'd a Counter, he did not diminish their Number; but if he miss'd of a white one, put it in again, and left an equal Hazard to him who had the following Choice; for if it be otherwise suppos'd, A's share will be $\frac{55}{123}$, which is less than $\frac{9}{19}$.

Prob. 2. It is evident, that wagering to pick out 4 Cards out of 40, so that there be one of every Suit, is no more, than wagering,

gering, out of 39 Cards to take 3 which shall be of three proposed Suits; for it is all one which Card you draw first, all the Hazard being, whether out of the 39 remaining you take 3, of which none shall be of the Suit you first drew. Suppose then you had gone right for three times, and were to draw your last Card, it is clear, that there are 27 Cards, (*viz.* of the Suits you have drawn before) of which, if you draw any you lose, and 10 of which, if you draw any, you have the Stake a ; so you have 10 Chances for a , and 27 for nothing, which, by Prop. 3. is worth $\frac{10}{37}a$. Suppose

D 3

54 *Solution of the*

pose again you had gone right only for two Draughts, then you have 18 Cards (of the Suits you have drawn before) which make you lose, and 20, which put you in the Case suppos'd formerly, *viz.* where you have but one Card to draw, which, as we have already calculated, is worth $\frac{10}{37}a$; so you have 18 Chances for nothing, and 20 for $\frac{10}{37}a$, which, by Prop. 3. is worth $\frac{100}{703}a$. Suppose again you have 3 Cards to draw, then you have 9 (of the Suit you drew first) which make you lose, and 30 which put you in the Case suppos'd last; so you have 9 Chances for nothing, and 30 for $\frac{100}{703}a$, which, by Propos.

Prop. 3. is worth $\frac{3000}{27417}a$, or $\frac{1000}{9139}a$, and you leave to your Fellow-Gamester $\frac{8139}{9139}a$; so your Hazard is to his as 1000 to 8139.

It is easie to apply this Method to the Games that are in use amongst us: For *Example*, If A and B, playing at *Backgammon*, B had already gain'd one end of three, and A none, and if A had the Dice in his Hand for the last Throw of the second end, all his Men but two upon the Ace Point being already cast of: *Quær.* What is the proportion of A's Hazard to B's?

Solution: There being of the 36 Throws of two Dice, six which give Doublets; if A
D 4. throw.

throw any of the Six, he has the Stake a ; if he throw any of the other Thirty, then he wants but one Game, and his Fellow-Gamester three, which, by Prop. V. is worth $\frac{7}{8} a$; so A has six Chances for a , and thirty for $\frac{7}{8} a$, which, by Prop. 3. is worth $\frac{129}{144} a$, and there remains to his Play-Fellow $\frac{15}{144} a$; so A's Hazard to B's, is as 129 to 15, that is, less than 9 to 1.

Supposing the same Case, and if their Bargain had been, that he who gain'd three ends before the other gain'd one, should have double of what each stak'd, that is, the Stake and a half more, then there had been due to A $\frac{281}{288}$ of the Stake,

Stake, that is, B ought only to take $\frac{1}{16}$, and leave the rest to A.

Thus likewise, if you apply the former Rule to the *Royal Oak-Lottery*, you will find, that he who wagers that any Figure shall come up at the first throw, ought to wagers 1 against 31; that he who wagers it shall come up at one of two throws, ought to wager 63 against 961; that he who wagers that a Figure shall come up at once in three times, ought to lay 124955 against 923621, &c. it being only somewhat tedious to calculate the rest. Where you will find, that the equality will not fall as some imagin on 16 Throws, no more than the e-

D 5 quality

quality of wagering at how many Throws of one Dye 6 shall come up, falls on three; the contrary of which you have seen already demonstrated; you will find by calculation, that he has the Disadvantage, who wagers, that 1 of the 32 different Throws of the *Royal Oak-Lottery*, shall come at once of 20 times, and that he has some Advantage, who wagers on 22 times; so the nearest to Equality is on 21 times: But it must be remembred, that I have suppos'd in the former Calculation, the Ball in the *Royal Oak-Lottery* to be regular, tho it can never be exactly so; for he who has the smallest Skill in
Geome.

Geometry, knows, that there can be no regular Body of 32 fides, and yet this can be of no advantage to him who keeps it.

*To find the Value of the
Throws of Dice as to the
Quantity.*

NOthing is more easie, than by the former Method to determine the Value of any Number of Throws of any Number of Dice; for in one Throw of a Dye, I have an equal chance for 1, 2, 3, 4, 5, 6, consequently my Hazard is worth

worth their Sum 21 divided by their Number 6, that is, $3\frac{1}{2}$. Now if one Throw of a Dye be worth $3\frac{1}{2}$, then two Throws of a Dye, or one Throw of two Dice is worth 7, two Throws of two Dice, or one Throw of four Dice is worth 14, &c. The general Rule being to multiply the Number of Dice, the Number of Throws, and $3\frac{1}{2}$ continually.

This is not to be understood as if it were an equal Wager to throw 7, or above it, with two Dice at one Throw; for he who undertakes to do so, has the advantage by 21 against 15. The meaning is only, if I were to have a Guinea, a Shilling,

Shilling, or any thing else, for every Point that I threw with two Dice at one Throw, my Hazard is worth 7 of these, because he who gave me 7 for it, would have an equal probability of gaining or losing by it, the Chances of the Throws above 7 being as many, as of these below it : So it is more than an equal Wager to throw 14 at least at two Throws of two Dice, because it is more probable that 14 will come, than any one Number besides, and as probable that it will be above it as below it; but if one were to buy this Hazard at the rate above-mention'd, he ought just to give 14 for it.

The

The equal Wager in one Throw of two Dice, is to throw 7 at least one time, and 8 at least another time, and so *per vices*: The reason is, because in the first Case I have 21 Chances against 15, and in the second 15 Chances against 21.

Of RAFFLING.

IN Raiffing the different throws
and their Chances are these ;

Where it is to be
observed, that of
the 216 different
Throws of three
Dice, there are on-
ly 96 that give
Doublets, or two,
at least, of a kind ;
so it is 4 to 5 that
with three Dice

<i>Throws.</i>	<i>Chau.</i>
3	18
4	17
5	16
6	15
7	14
8	13
9	12
10	11
	9

you shall throw Doublets, and
it is 1 to 35 that you throw a
Raffle, or all three of a kind.

it

It is evident likewise, that it is an even Wager to throw 11 or above it, because there are as many Chances for 11, and the Throws above it, as for the Throws below it; but tho it be an even Wager to throw 11 at one Throw, it is a disadvantage to wager to throw 22 at two Throws, and far more to wager to throw 33 at three Throws; and yet it is more than an equal Wager that you shall throw 21 at two Throws in Raffling, because it is as probable that you will, as that you will not throw 11, at least, the first time, and more than probable that you will throw 10, at least, the second time.

For

Hazards of Game. 65

For an instance of the plainness of the preceeding Method, I will shew, how by simple Subtraction, the most part of the former Problems may be solv'd.

Suppose A and B, playing together, each of 'em stakes 32 Shillings, and that A wants one Game of the Number agreed on, and B wants two; to find the share of the Stakes due to each of 'em. It's plain, if A wins the next Game he has the whole 64 Shillings; if B wins it, then their Shares are equal; therefore says A to B, If you will break off the Game, give me 32, which I am sure of, whether I win or lose the
next

next Game, and since you will not venture for the other 32, let us part them equally, that is, give me 16, which, with the former 32, make 48, leaving 16 to you.

Suppose A wanted one Game, and B three; if A wins the next Game, he has the 64 Shillings; if B wins it, then they are in the condition formerly suppos'd, in which Case there is 48 due to A; therefore says A to B, give me the 48 which I am sure of, whether I win or lose the next Game, and since you will not hazard for the other 16, let us part them equally, that is, give me 8, which, with the former 48, make 56, leaving
ing

ing 8 to you, and so all the other Cases may be solv'd after the same manner.

Suppose A wagers with B, that with one Dye he shall throw 6 at one of three Throws, and that each of them stakes 108 Guineas: To find what is the proportion of their Hazards; Now there being in one Throw of a Dye but one Chance for 6, and five Chances against it, one Throw for 6 is worth $\frac{1}{6}$ of the Stake; therefore says B to A, of the 216 Guineas take a sixth part for your first Throw, that is, 36; for your next Throw take a sixth part of the remaining 180, that is, 30; and for your third Throw,

Throw, take a sixth part of the remaining 150, that is, 25, which in all make 91, leaving to me 125; so his Hazard who undertakes to throw 6 at one of three Throws, is 91 to 125.

Suppose A had undertaken to throw 6 with one Dye at one Throw of four, and that the whole Stake is 1296; says A to B, Every Throw for 6 of one Dye, is worth the sixth part of what I throw for; therefore for my first Throw give me 216, which is the sixth part of 1296, and there remains 1080, I must have the sixth part of that, viz. 180, for my second Throw; and the sixth
part

part of the remaining 900, which is 150, for my third Throw; and the sixth part of the last remainder 750, which is 125 for my fourth Throw; all this added together makes 671, and there remains to you 625; so it is evident, that A's Hazard, in this Case, is to B's 671 to 625.

Suppose A is to win the Stakes (which we shall suppose to be 36) if he throws 7 at once or twice with two Dice, and B is to have them if he does not; says B to A, the Chances which give 7 are 6 of the 36, which is as much as 1 of 6; therefore for your first Throw you shall have a sixth part

part of the 36, which is 6; and for your next Throw a sixth part of the remainder 30, which is 5; this in all makes 11; so you leave 25 to me; so A's Hazard is to B's as 11 to 25.

It were easie, at this rate to calculate the most intricate Hazards, were it not that Fractions will occur, which, if they be more than $\frac{1}{2}$, may be suppos'd equal to an Unit, without causing any remarkable Error in great Numbers.

It will not be amiss, before I conclude, to give you a Rule for finding in any Number of Games the Value of the first, because *Hugens's* Method, in
that

that Case, is something tedious.

Suppose A and B had agreed, that he should have the Stakes who did win the first 9 Games, and A had already won one of the 9; I would know what share of B's Mony is due to A for the Advantage of this Game. To find this, take the first eight even Numbers 2, 4, 6, 8, 10, 12, 14, 16, and multiply them continually; that is, the first by the second, the product by the third, &c. take the first eight odd Numbers, 1, 3, 5, 7, 9, 11, 13, 15, and do just so by them, the product of the even Number is the Denominator, and the product of the

the odd Number the Numerator of a Fraction, which expresseth the quantity of B's Money due to A upon the winning of the first Game of 9; that is, if each stak'd a number of Guineas, or Shillings, &c. express'd by the product of the even Numbers, there would belong to A, of B's Money, the Number express'd by the product of the odd Numbers: For *Example*, Suppose A had gain'd one Game of 4, then by this Rule, I take the three first even Numbers, 2, 4, 6, and multiply them continually, which make 48, and the first three odd Numbers, 1, 3, 5, and multiply them continually, which

which make 15 ; so there belongs to A $\frac{15}{48}$ of B's Money, that is, if each stak'd 48, there would belong to A, besides his own 15 of A's. Now by *Hugens's* Method, if A wants but three Games while B wants four, there is due to A $\frac{21}{32}$ of the Stake ; by this Rule there is due to A $\frac{15}{48}$ of B's Money, which is $\frac{15}{69}$ of the Stake, which, with his own $\frac{48}{96}$ of the Stake, makes $\frac{63}{96}$ or $\frac{21}{32}$ of the Stake, and so in every Case you will find *Hugens's* Method and this will give you the same Number ; a Demonstration of it you may see in a Letter of Monsieur *Pascals* to Monsieur *Fermat* ; tho it be otherwise express'd there than here,

E

yet

yet the consequence is easily supply'd. To prevent the labour of Calculation, I have subjoyn'd the following Table, which is caloulated for two Gamesters, as *Monf. Hugen* is for three.

If each of us stake 256 *Guineas*
in

There belongs to me of 256 of my Play-Yellow		6	5	4	3	2	1
	1st. Game	63	70	80	96	128	256
	2 1st. Games	126	140	160	192	256	
	3 1st. Games	182	200	224	256		
	4 1st. Games	224	240	256			
	5 1st. Games	248	256				
	6 1st. Games	256					

The

The Use of the Table is plain; for let our Stakes be what they will, I can find the Portion due to me upon the winning the first, or the first two Games, &c. of 2, 3, 4, 5, 6. For Example, If each of us had stak'd 4 Guineas, and the Number of Games to be plaid were 3, of which I had gain'd 1, say, As 256 is to 96, so is 4 to a fourth.

$$256 : 96 :: 4 : 1\frac{1}{2}$$

To find what is the Value of his Hazard, who undertakes, at the first Throw, to cast Doublets, in any given Number of Dice.

In two Dice it is plain to avoid Doublets, every one of the six different Throws of the first, can only be combin'd with five of the second, because one of the six is of the same kind, and consequently makes Doublets ; for the same reason, the thirty Throws of two Dice, which are not Doublets, can only be combin'd with four Throws of a third Dice, and three Throws of a fourth Dice ; so generally it is this Series,

$$6 \times 5 \times 4 \times 3 \times 2 \times 1 \times 0, \text{ \&c.}$$

$$6 \times 6 \times 6 \times 6 \times 6 \times 6 \times 6, \text{ \&c.}$$

The second Series is the Sum of the Chances, and the first the
Number

Number of Chances against him who undertakes to throw Doublets, each Series to be continu'd so many terms, as are the Number of Dice. For *Example*, If one should undertake to throw Doublets at the first Throw of four Dice, his Adversary's Ha-

zard is $\frac{6 \times 5 \times 4 \times 3}{6 \times 6 \times 6 \times 6} = \frac{360}{1296}$ or

$\frac{5}{18}$ leaving to him $\frac{13}{18}$, so he

has 13 to 5. In seven Dice, you see the Chances against him are 0, because then there must necessarily be Doublets.

Of WHIST.

IF there be four playing at Whist, it is 15 to 1 that any two of them shall not have the four Honours, which I demonstrate thus:

Suppose the four Gamesters be A, B, C, D: If A and B had, while the Cards are a dealing, already got three Honours, and wanted only one, since it is as probable that C and D will have the next Honour, as A and B; if A and B had laid a Wager to have it, there is due to them but $\frac{1}{2}$ of the Stake: If A and B wanted

wanted two of the four, and had wager'd to have both those two, then they have an equal Hazard to get nothing; if they miss the first of those two, or to put themselves in the former Case if they get it; so they have an equal Hazard to get nothing or $\frac{1}{2}$, which, by Prop. 1. is worth $\frac{1}{4}$ of the Stake; so if they want three Honours, you will find due to them $\frac{1}{8}$ of the Stake; and if they wanted four, $\frac{1}{16}$ of the Stake, leaving to C and D $\frac{15}{16}$; so C and D can wager 15 to 1, that A and B shall not have all the four Honours.

It is 11 to 5 that A and B shall not have three of the four
E 4 Ho-

Honours , which I prove thus :

It is an even Wager, if there were but three Honours, that A and B shall have two of these three, since 'tis as probable that they will have two of the three, as that C and D shall have them ; consequently, if A and B had laid a Wager to have two of three, there is due to them $\frac{1}{2}$ of the Stake. Now suppose A and B had wager'd to have thræe of four, they have an equal Hazard to get the first of the four, or miss it ; if they get it, then they want two of the three, and consequently there is due to them $\frac{1}{2}$ of the Stake ; if they miss it, then they want three of the three, and consequently there

Hazards of Game, 81

there is due to them $\frac{1}{8}$ of the Stake; therefore, by Prop. 1. their Hazard is worth $\frac{5}{16}$, leaving to C and D $\frac{11}{16}$.

A and B playing at Whist against C and D; A and B have eight of ten, and C and D nine, and therefore can't reckon Honors; to find the proportion of their Hazards.

There is $\frac{5}{16}$ due to C and D upon their hazard of having three of four Honours; but since A and B want but one Game, and C and D two, there is due to C and D but $\frac{1}{4}$, or $\frac{4}{16}$ more upon that account, by Prop. 4. this in all makes $\frac{9}{16}$,
E 5 leaving

leaving to A and B $\frac{7}{16}$; so the hazard of A and B to that of C and D, is as 9 to 7.

In the former Calculations I have abstracted from the small difference of having the Deal and being Seniors.

All the former Cases can be calculated by the *Theorems* laid down by Monsieur *Hugens*; but Cases more compos'd require other Principles, for the easie and ready Computation of which, I shall add one *Theorem* more, demonstrated after Mons. *Hugens's* Method.

Theorem.

Theor.

If I have p Chances for a , q Chances for b , and r Chances for c , then my hazard is worth $\frac{ap + bq + cr}{p + q + r}$, that is, a multiplied into the number of its Chances added to b , multiplied into the number of its Chances, added to c multiplied into the number of its Chances, and the Sum divided by the Sum of Chances of a, b, c .

To investigate as well as demonstrate this Theorem, suppose the value of my hazard be x , then x must be such, as having it, I am able to purchase as good

good a hazard again in a just and equal Game. Suppose the Law of it be this, That playing with so many Gamesters as, with my self, make up the number $p+q+r$, with as many of them as the number p represents; I make this bargain, that whoever of them wins shall give me a , and that I shall do so to each of them if I win; with the Gamesters represented by the number of q , I bargain to get b , if any of them win, and to give b to each of them, if I win my self; and with the rest of the Gamesters, whose number is $r-1$, I bargain to give, or to get c after the same manner: Now all being in an
equal

equal probability to gain, I have p Chances to get a , q Chances to get b , and $r-1$ Chances to get c , and one Chance, *viz.* when I win myself, to get $px + qx + rx - ap - bq - rc + c$, which, if it be suppos'd equal to c , then I have p Chances for a , q Chances for b , and r Chances for c (for I had just now $r-1$ Chances for it) therefore, if $px + qx + rx - ap - bq - rc + c = c$, then is $x = \frac{ap + bq + cr}{p + q + r}$.

By the same way of reasoning you will find, if I have p Chances for a , q Chances for b , r Chances for c , and s Chances

ces for d , that my hazard is

$$\frac{ap + bq + cr + ds}{p + q + r + s}, \text{ \&c.}$$

In Numbers.

If I had two Chances for 3 Shillings, four Chances for 5 Shillings, and one Chance for 9 Shillings, then, by this Rule, my hazard is worth 5 Shillings;

$$\text{for } \frac{2 \times 3 + 4 \times 5 + 1 \times 9}{7} = 5; \text{ and}$$

it is easie to prove, that with 5 Shillings I can purchase the like hazard again; for suppose I play with six others, each of us staking 5 Shillings; with two of them I bargain, that if either of them win, he must give

give me 3 Shillings, and that I shall do so to them; and with the other four I bargain just so, to give or to get 5 Shillings: This is a just Game, and all being in an equal probability to win; by this means I have two Chances to get 3 Shillings, four Chances to get 5 Shillings, and one Chance to get 9 Shillings, *viz.* when I win my self; for then out of the Stake, which makes 35 Shillings, I must give the first two 6 Shillings, and the other four 20 Shillings, so there remains just 9 to my self.

It is easie, by the help of this *Theorem*, to calculate in the Game of Dice, commonly call'd
Hazard,

Hazard, what Mains are best to sett on, and who has the Advantage, the Caster or Setter. The Scheme of the Game, as I take it, is thus,

<i>Mains.</i>	Throws next following for	
	The Caster.	The Setter.
V.	V.	II. III. XI. XII.
VI.	VI. XII.	XI. II. III.
VII.	VII. XI.	XII. II. III.
VIII.	VIII. XII.	XI. II. III.
IX.	IX.	II. III. XI. XII.

By an easie Calculation you will find, if the Caster has VI. and the Setter VII, there is due to the Caster $\frac{1}{3}$ of the Stake; if he has

V. against VII. $\frac{2}{5}$ of the Stake,
 VI. against VII. $\frac{5}{11}$ of the Stake,
 IV. against VI. $\frac{3}{8}$ of the Stake,
 V. against VI. $\frac{4}{9}$ of the Stake,
 VI. against V. $\frac{3}{7}$ of the Stake,

I need not tell the Reader,
 that IV. is the same with X,
 V. with IX, and VI. with VIII.

Suppose then VII. be the
 Main: To find the propor-
 tion of the hazard of the Ca-
 ster to that of the Setter.

By the Law of the Game,
 the Caster, before he throws
 next, has four Chances for no-
 thing, *viz.* these II, III, XII;
 eight Chances for the whole
 Stake, *viz.* those of VII, XI;
 six

six Chances for $\frac{1}{3}$, viz. those IV, X; eight Chances for $\frac{2}{5}$, viz. those of V, IX; and ten Chances for $\frac{5}{11}$, viz. those of VI, X; so his hazard, by the preceding Theorem, is

$$\frac{4 \times 0 + 8 \times 1 + 6 \times \frac{1}{3} + 8 \times \frac{2}{5} + 10 \times \frac{5}{11}}{36}$$

Now to save the trouble of a tedious reduction, Suppose the Stake which they play for be 36, that is, the Setter had laid down 18; in that case, every one of these Fractions are so many parts of an Unite, which, being gather'd into one Sum, give $17\frac{41}{55}$ to the Caster, leaving $18\frac{14}{55}$ to the Setter; so the hazard of the Caster is to that of the Setter 244, 251. Sup-

Suppose VI. or VIII. be the Main, then the Share of the Caster is

II.

III. VI. IV. V.

XI. XII. X. IX. VIII. VII.

$5x_0 + 6x_1 + 6x_{\frac{1}{2}} + 8x_{\frac{2}{3}} + 5x_{\frac{1}{2}} + 5x_{\frac{1}{11}} =$
 $= 17\frac{229}{396}$, leaving to the Setter
 $18\frac{167}{396}$, so the hazard of the Ca-
 ster is to that of the Setter as
 6961 to 7295.

Suppose V. or IX. be the Main, then the Share of the Caster is

II.

III.

XI.

IV.

VI.

XII. V. X. IX. VIII. VII.

$6x_0 + 4x_1 + 6x_{\frac{1}{2}} + 4x_{\frac{1}{2}} + 10x_{\frac{2}{3}} + 6x_{\frac{1}{11}} =$
 $= 17\frac{229}{315}$, leaving to the Set-

ter

ter is $18\frac{86}{315}$, so the hazard of the Caster is to that of the Setter as 1396 to 1493.

It is plain, that in every Case the Caster has the Disadvantage, and that V. or IX. are better Mains to set on than VII, because, in this last Cast the Setter has but 18 and $\frac{14}{55}$ or $\frac{84}{330}$; whereas, when V. or IX. is the Main, he has $18\frac{86}{315}$; likewise VI. or VIII. are better Mains than V. or IX. because $\frac{167}{396}$ is a greater Fraction than $\frac{86}{315}$.

All those Problems suppose Chances, which are in an equal probability to happen, if it should

should be suppos'd otherwise, there will arise variety of Cases of a quite different nature, which, perhaps, 'twere not unpleasant to consider, I shall add one *Problem* of that kind, leaving the *Solution* to those who think it merits their pains.

In Parallelipipedo cujus latera sunt adinvicem in ratione a, b, c: Invenire quotâ vice quisvis suscipere potest, ut datum quodvis planum, v. g. ab jaciât.

F I N I S.

E R R A T A.

PReface, page 3. line 1. read *in*. p. 6. l. 5. r. *incur*. p. 10. l. 8. for *is left to me*, r. *properly deserves the name of Condu&*. Book, p. 2. l. 7. for 9 r. 7. p. 16. l. 5. add *and he one*. p. 71. l. 5. r. *wins*.

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